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Hadronic decays from the lattice

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Abstract. I review the lattice QCD approach to determining hadronic-decay transitions. Examples considered include $\rho \to \pi\pi$; $b_1 \to \pi\omega$; hybrid meson decays and scalar meson decays. I discuss what lattices can provide to help understand the composition of hadrons.

PACS. 13.25.-k Hadronic decays of mesons – 12.38.Gc Lattice QCD calculations – 12.39.Mk Glueball and nonstandard multi-quark/gluon states

1 Introduction

Relatively few hadronic states are stable to strong decays (*i.e.*, via QCD with degenerate u and d quarks). Among the mesons, we have [1]

Stable:	$\pi,\ K,\ \eta,\ D,\ D_s,\ B,\ B_s,\ B_c,$ $D_s^*,\ B^*,\ B_s^*,\ D_s(0^+),\ B_s(0^+)$;
$\Gamma < 1\,\text{MeV}$:	$\eta',\ D^*,\ \psi(1S),\ \psi(2S),\ \chi_1,\ \chi_2,$ $\Upsilon(1S),\ \Upsilon(2S),\ \Upsilon(3S)$;
$\Gamma < 10\,\text{MeV}$:	$\omega,\ \phi,\ \chi_0,\ X(3872)$;
$\Gamma > 10\,\text{MeV}$:	$\rho,\ f_0,\ a_0,\ h_1,\ b_1,\ a_1,\ f_2,\ f_1,\ a_2,$ etc., including η_c.

The mass of an unstable state is usually defined as the energy corresponding to a 90° phase shift. This definition[1] seems to accord with simple mass formulae: For example,

- $\rho(776)$ and $\omega(783)$ are close in mass despite having widths of 150 and 8 MeV, respectively.
- The baryon decuplet ($\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$) is roughly equally spaced in mass despite having widths of (120, 37, 9, 0) MeV, respectively.

So, on the one hand, unstable particles seem to fit in well with stable ones; on the other hand, the presence of open decay channels will have an influence in lattice studies.

Some of the motivations to study hadronic decays on the lattice are:

- What are hadrons made of?
  Is a meson made predominantly of $\bar{q}q$, $\bar{q}\bar{q}qq$ or meson-meson?
- A state that can decay strongly (resonance) necessarily has a meson-meson component —is this important?
  Are unstable particles different from stable ones?
- What is the nature of light-light scalar mesons?
  Where is the glueball?
- Are there hybrid mesons?

Lattice QCD is a first-principles method of attack. But we use unphysical (too heavy) quark masses, we have Euclidean time. So what can we learn?

2 Decays in Euclidean time

NO GO. At large spatial volume, the two-body continuum *masks* any resonance state. The extraction of the spectral function from the correlator $C(t)$ is ill-posed unless a model is made [3,4], since the low-energy continuum dominates at large t.
GO. For finite spatial volume (L^3), the two-body continuum is *discrete* and Lüscher showed [5–7] how to use the small energy shifts with L of these two-body levels to extract the elastic-scattering phase shifts. The phase shifts then determine the resonance mass and width, see ref. [8] for a review. Thus, a relatively broad resonance such as the ρ appears as a distortion of the $\pi_n\pi_{-n}$ energy levels where the pion momentum $q = 2\pi n/L$.

As a check of this approach to unstable particles on a lattice, the coupling of ρ to $\pi\pi$ has been determined from first principles [9]. The method is to arrange the ρ and $\pi\pi$ state (with definite relative momentum) to be approximately degenerate in energy on a lattice. Then, several independent methods allow to determine the transition amplitude x and, hence, the effective coupling constant $\bar{g}$ from the lattice (where decay does not proceed) and compare with experiment:

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[1] Note that defining the mass as the real part of the pole will cause a downward shift of masses for wider states, *e.g.*, 22 MeV less [2] for the $\Delta(1232)$ pole, and this prescription will fit the equal mass rule less well.

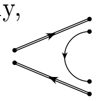
Method	m_{val}	m_{sea}	$\bar{g}$
Lattice xt	s	s	1.40^{+47}_{-23}
Lattice ρ shift	s	s	1.56^{+21}_{-13}
$\phi \to K\bar{K}$	s	$u,\ d$	1.5
$K^* \to K\pi$	$u,\ d/s$	$u,\ d$	1.44
$\rho \to \pi\pi$	$u,\ d$	$u,\ d$	1.39

Note that the lattice has heavier sea quarks than experiment. Nevertheless, the level of agreement between first-principles lattice evaluation and experiment is very encouraging.

2.1 Hadronic transitions from the lattice

Here I summarise the steps that allow a fairly direct determination of hadronic transition amplitudes from the lattice.

i) Consider a lattice study of the off-diagonal correlator: from a ρ-meson to $\pi\pi$. Diagrammatically,



$\rho \to \pi\pi$
0 ———–X ———–0
0 t T

ii) Now to evaluate this contribution, since the intermediate point marked X at time t is not observed on a lattice, it must be summed over:

$$\sum_{t=0}^{T} e^{-m(\rho)t} x e^{-m(\pi\pi)(T-t)} \to xTe^{-mT}$$

if $m(\rho) \approx m(\pi\pi)$.

iii) So a plot of the (normalised) transition from the lattice *versus* T has slope of x, which can be related [9] to the continuum coupling g^2.

Note this approach is like measuring the wave function overlap —but with no model for the wave functions. In order to control excited-state contributions, however, it is subject to the restriction that initial and final states have similar energies on a lattice. A more rigorous approach is possible by using the method of determining the two-body energy *versus* lattice volume, described above, but in practice sufficient precision is not available in general to study resonance decays, although results for scattering lengths have been obtained.

3 Hybrid meson decay

One of the characteristic predictions of QCD is that there can be mesons in which the gluonic degrees of freedom are non-trivially excited. The simplest example is a hybrid meson with spin-exotic $J^{PC} = 1^{-+}$ which is a J^{PC} combination not available to a $\bar{q}q$ state. The spin-exotic quantum numbers then require a non-trivial gluonic contribution to the state. These hybrid mesons have been studied extensively on the lattice, here I discuss their decay.

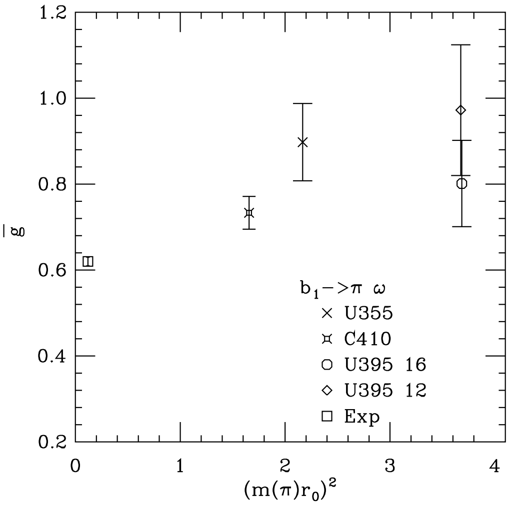


Fig. 1. First test of S-wave decays from the lattice for $b_1 \to \pi\omega$ from ref. [10]. The effective coupling $\bar{g}$ is plotted *versus* the quark mass, as determined by the pseudoscalar mass squared where the scale is set by $r_0 \approx 0.5\,\text{fm}$.

3.1 Light quarks

The S-wave decay of the $J^{PC} = 1^{-+}$ spin-exotic hybrid meson($\hat{\rho}$) to πb_1 has been studied recently on the lattice [10]. Since S-wave decays have not previously been studied in this way, a check was made by extracting the decay strength for the S-wave component of the transition $b_1 \to \pi\omega$. As shown in fig. 1, the lattice determination, though using heavier quarks than experiment, fits in well. This gives confidence that the hybrid decay prediction will be reliable. Since the experimental effective coupling constant lies lower than the lattice results, this also shows that lattice results, with heavier than experimental quark masses, may overestimate the coupling somewhat. This may be interpreted, phenomenologically, as arising from form factor effects [11].

From lattices with $N_f = 2$ flavours of sea quark, a recent study [10] obtains a spin-exotic hybrid meson state at 2.2(2) GeV. The S-wave decay transitions are then evaluated, as shown in fig. 2, obtaining a partial width to πb_1 of 400(120) MeV and to πf_1 of 90(60) MeV. These results indicate that the decay width of this hybrid meson may be large and, hence, more difficult to extract experimentally.

For a recent preliminary lattice study of decay of a $J^{PC} = 1^{-+}$ hybrid meson to πa_1 using the Lüscher method which obtains a width of around 60 MeV, see ref. [12]. This decay channel implies that the hybrid meson considered has $I = 0$, rather than $I = 1$ as above.

3.2 Heavy quarks

The cleanest environment in which to study such hybrid states on a lattice is in the limit of very heavy quarks which is relevant to $\bar{b}b$. This can be approximated by using static quarks and the gluonic excitation arises as an

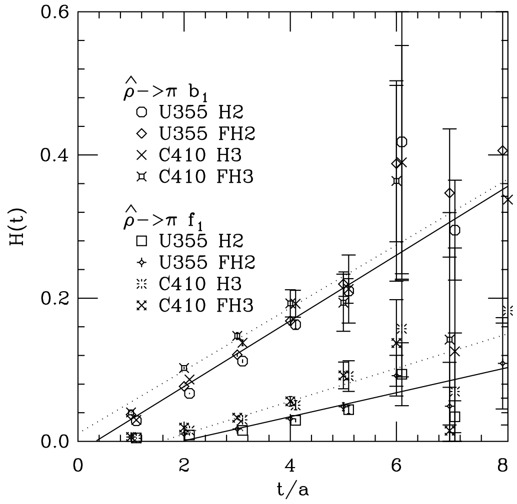


Fig. 2. Spin-exotic hybrid decay transition from ref. [10]. Here the decay transition strength is given by the slope.

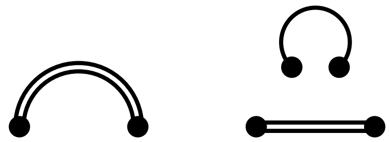


Fig. 3. Initial and final states relevant for the decay of the heavy-quark spin-exotic hybrid meson to $\chi_b f_0$.

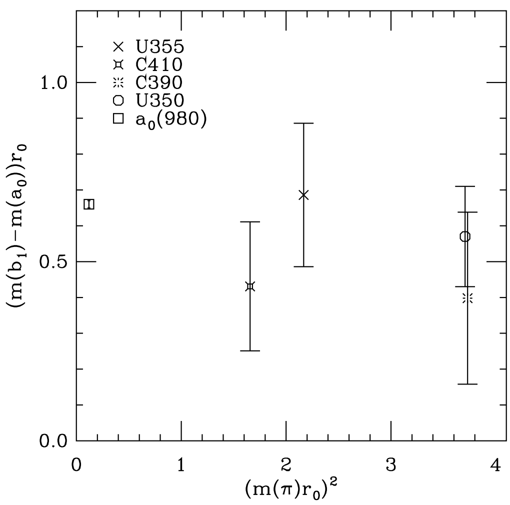


Fig. 4. Mass difference of b_1 and a_0 mesons from ref. [19] plotted *versus* the quark mass, as determined by the pseudoscalar mass squared with the scale is set by $r_0 \approx 0.5\,\mathrm{fm}$. The open box is the experimental point if the a_0-meson is at 980 MeV.

excited string state between these static quarks with non-trivial gluonic angular momentum. Lattice studies have long predicted the spectrum of such states.

To guide experiment, however, it is important to know the expected decay mechanism and associated width. In the static quark limit, several symmetries can be used which imply [13] that the dominant decay will be string de-excitation (rather than string breaking) as illustrated in fig. 3. Lattice study [13] shows that the dominant decay of the hybrid meson H_b is string de-excitation to $\chi_b f_0$. The width is predicted to be around 80 MeV.

This estimate from first principles of the decay width is of significance in guiding experimental searches for such hybrid states.

4 Scalar mesons

4.1 Light quarks

Since $u\overline{u} + d\overline{d}$, $s\overline{s}$, glueball, and meson-meson components are all possible for flavour-singlet scalar mesons, this is a difficult area to study both on a lattice, and in interpreting experimental data. For scalar mesons the lowest mass decay channels are $\pi\pi$ (flavour singlet: f_0) or $\eta\pi$ (flavour non-singlet: a_0) and these decay channels are open in many dynamical lattice studies. The history of lattice attempts to study the complex mixing between these different contributions is

- 0^{++} glueball decay $\to \pi\pi$: quenched study [14,15].
- Glueball mixing with $q\overline{q}$-meson. This hadronic transition has been studied using quenched [16] and dynamical lattices [17].

A full lattice study is needed which includes glueball, $\overline{q}q$ and $\pi\pi$ channels but the disconnected diagram for $f_0 \to \pi\pi$ is very noisy in practice —as shown in ref. [18].

To reduce the contribution from disconnected diagrams, one can study flavour non-singlet scalar mesons. The simplest case is a_0 which has a decay $\eta\pi$ and this has been explored in quenched studies which have an anomalous behaviour: since the η itself is unphysical (appearing as a double pole degenerate in mass with the pion). Rather than try to correct for this anomaly which gives a wrong sign to the a_0 correlator at larger t, it is preferable to use a ghost-free theory. With two flavours of sea quark ($N_f = 2$), this problem is avoided. A recent study [19] of the a_0-meson concentrates on the mass difference between it and the b_1-meson, as shown in fig. 4. This study concludes that the $\bar{q}q$ non-singlet scalar meson lies around 1 GeV —which is considerably lighter than some previous lattice studies (see ref. [19] for a summary).

As well as determining the mass values, this study evaluates the $a_0 \to \eta\pi$ and $a_0 \to KK$ transitions. Results from the connected contribution to these transitions are shown in fig. 5. The resulting coupling constant is determined to be of similar value to that obtained by some phenomenological studies of decays of the $a_0(980)$-meson. This again points to the possibility that the $a_0(980)$ may be substantially a $\bar{q}q$ state.

A full study of flavour singlet scalar mesons will need to take into account the $\bar{q}q$, gluonic and meson-meson channel and their mixing. This has not yet been achieved, for a summary of the current state of lattice studies see ref. [18].

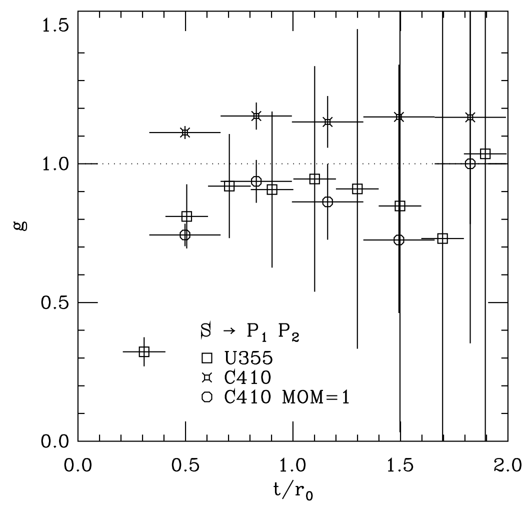


Fig. 5. Effective coupling from connected contribution to decay transition of scalar meson → 2 pseudoscalars from ref. [19]. The t region around 0.5 to 1.5 is expected to be relevant (in units with $r_0 \approx 0.5\,\mathrm{fm}$).

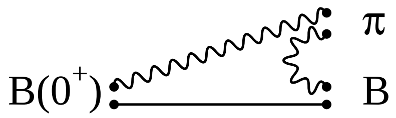


Fig. 6. Diagram for the $B(0^+) \to B\pi$ transition.

4.2 Heavy-light quarks

One of the most promising ways to study scalar mesons on lattice is through heavy-light mesons. The scalar meson with $\bar{c}s$ quantum number is known experimentally [1] to be very narrow (it decays only via the isospin-violating channel $D_s\pi$ or electromagnetically), while the scalar meson with $\bar{b}s$ quantum number is predicted to be similarly narrow from a lattice study of its energy [20].

Here we consider the heavy-light scalar meson in the limit of a static heavy quark —which will be relevant to mesons with a b-quark. Lattice studies indicate [20] that the $\bar{b}s$ scalar meson is expected to be stable to strong decay while the $\bar{b}n$ scalar meson (where $n = u, d$) will decay to $B\pi$. Evaluating the diagram shown in fig. 6, a lattice estimate, shown in fig. 7, of the decay rate of $B(0^+) \to B(0^-)\pi$ gives a width predicted [21] as 162(30) MeV. This state has not been observed experimentally yet, but the experimental results for the corresponding $\bar{c}n$ state, $D(0^+)$, are that the width is $270 \pm 50\,\mathrm{MeV}$. Although significant $1/m_Q$ effects are expected in the HQET in extrapolating to charm quarks, this is indeed a similar magnitude to that predicted for B mesons. It will be interesting so see how the lattice prediction of the mass and width of the $B(0^+)$ fares when experimental results are available.

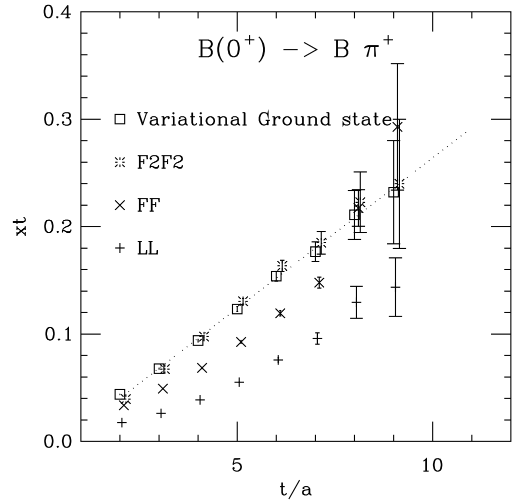


Fig. 7. $B(0^+) \to B\pi$ transition strength (given as slope) on a lattice from ref. [21].

5 Do decays matter?

The previous discussion of decays on a lattice emphasises that $q\bar{q}$ states do mix with two-body states with the same quantum numbers. In the real world, the two-body states are a continuum and nearby states have a predominant influence. Since there is a suppression in the amplitude near threshold from the factor q^L for an L-wave transition, for S-wave transitions ($L = 0$) the threshold will turn on most abruptly and hence will have stronger mixing.

For bound states there is an influence of nearby many-body states (*e.g.*, $N\pi$ on N or $\pi\pi\pi$ on π) which mix to reduce the mass. The nearest such thresholds will be those with pionic channels since pions are the lightest mesons. This is the province of low-energy effective theories, especially chiral perturbation theory which is discussed in other talks. This then provides a reliable guide in extrapolating lattice results to the physical light quark masses.

For unstable states (resonances) the influence of the two-body continuum is less clear since the two-body states are both lighter and heavier. In the continuum at large volume, effective field theories can again be used to explore this. On a lattice, however, the signal for a particle becomes obscured as the quark mass is reduced so that it becomes unstable. Techniques, such as those discussed above, are needed to extract the elastic-scattering phase shift and hence the mass and width.

For quenched QCD, however, where these two-body states are not coupled (or have the wrong sign as in $a_0 \to \eta\pi$), then the unstable states will be distorted compared to full QCD. For instance, in existing quenched QCD studies, the ρ will be *too heavy* since it is not repelled by the heavier $\pi\pi$ states. Indeed an example of this effect was seen above in the study of ρ decay including dynamical sea quarks, where the ρ mass decreased [9] when it could couple to $\pi\pi$ compared to when it could not.

6 Molecular states?

Can lattice QCD provide evidence about possible molecular states: hadrons made predominantly of two hadrons?

The prototype is the deuteron: $n\ p$ bound in a relative S-wave (with some D-wave admixture) by π exchange.

There are many states close to two-body thresholds. Since S-wave thresholds are the most abrupt, it is usually in this case that the influence of the threshold on the state has been discussed. Some of these cases are

$$\begin{aligned} f_0(980)\ a_0(980) &\leftrightarrow K\overline{K}, \\ D_s(0^+) &\leftrightarrow D(0^-)K, \\ B_s(0^+) &\leftrightarrow B(0^-)K, \\ X(3872) &\leftrightarrow D^*\overline{D}, \\ \Lambda(1405) &\leftrightarrow \overline{K}N, \\ N(1535) &\leftrightarrow \eta N. \end{aligned}$$

Some of these states ($D_s(0^+)$, $B_s(0^+)$) are stable (in QCD in the isospin-conserving limit) whereas the rest have other channels open. There is a large literature, stretching over 40 years, discussing the consequences of the nearby threshold on these states. One definite implication is that isospin breaking is enhanced by mass splittings in thresholds (*e.g.*, $\overline{K^0}K^0$ compared to K^+K^- is 8 MeV higher and this induces isospin mixing between the states at 980 MeV). This level of detail is not accessible in lattice studies at present, but lattice QCD should be able to address the issue of the influence of thresholds on these states.

The observation of a state near a 2-body threshold implies that there is an attractive interaction between the two bodies. But this is a topic like that of whether the chicken or egg was created first: an attractive interaction implies and is implied by a nearby state. What can lattice QCD offer here? We are in the position of being able to vary the quark masses and this is a very useful tool. A two-body threshold will move in general in a different way with changing quark mass than a $\overline{q}q$ state. We can also move the strange and non-strange masses separately and this can be helpful too.

Another line of investigation is that lattice studies can explore the wave function of a state —either the Bethe-Salpeter wave function or the charge or matter spatial distribution. One can also explore the coupling of a state to a 2-body channel, as was discussed above.

The prototype of a molecular state is the deuteron: it has a tiny binding energy (2.2 MeV) and a very extended spatial wave function. Pion exchange between neutron and proton gives a mechanism for this long-range attraction. In general, it is difficult to reproduce such small binding energies in lattice studies.

Another case where a long-range pion exchange can give binding is in the BB system. Here lattice results indicate [22] the possibility of molecular bound states in some quantum number channels which have an attractive interaction from pion exchange, but also the possibility of bound multi-quark states which are not described as hadron-hadron but where the two heavy quarks form a colour triplet and the light quarks are arranged as in a heavy-light-light baryon. This BB example illustrates the rich structure available to multi-quark systems.

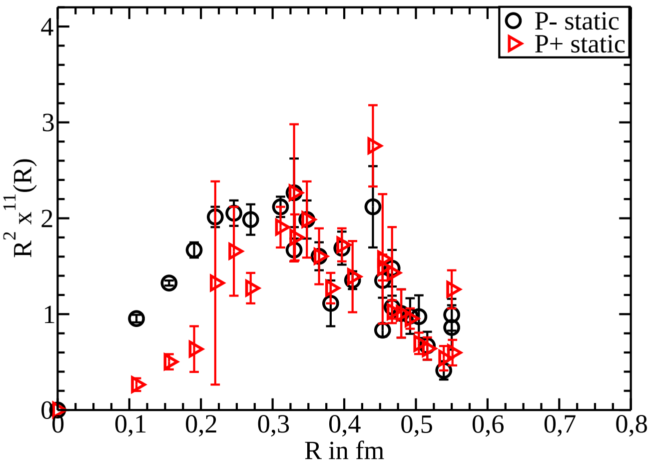


Fig. 8. Charge distributions of $B_s(0^+)$ and $B_s(2^+)$ mesons from the lattice. The heavy quark is static and the light quark is at a distance R from it. The $J^P = 0^+$ meson is P_-, while the $J^P = 2^+$ meson is P_+.

One case where lattice studies have been able to shed considerable light is for the $\overline{b}s$ and $\overline{c}s$ scalar mesons. Since, in the isospin-conserving limit, these states cannot couple to $B_s\pi$, $D_s\pi$, they have a lightest open decay channel BK, DK. Lattice studies [20,23] indicate that, in both cases, the scalar meson lies below the open threshold, so the states should be stable.

Because simple quark model expectations were that these scalar mesons were unstable, theorists have suggested that a BK, DK molecular composition was responsible. This can be explored on a lattice by measuring the spatial distribution of the heavy-light meson. A study [24] of the charge distribution of the light quark in a $B_s(0^+)$-meson is illustrated in fig. 8. This shows that the light quark spatial distribution is similar to that of other $\overline{Q}q$ states (*e.g.*, $J^P = 2^+$) for which no molecular interpretation is proposed. This reinforces the conclusion that the $B_s(0^+)$ is predominantly a $\overline{b}s$ state. The decay transition from $B_s(0^+)$ to BK has also been determined (see above) and it has an effective coupling constant which is consistent with that found for other (non-molecular) decays. Overall, lattice evidence does not support the hypothesis that $B_s(0^+)$ is a molecular state.

7 Conclusions

Lattice can address hadronic structure:

- Form factors (*e.g.*, charge wave functions) can be evaluated;
- Decay transitions (and mixing transitions) can be evaluated;
- Structure function moments can be evaluated (steady but slow progress here);
- Hadronic matrix elements are needed to interpret experiment (*e.g.*, f_B relates the B-meson to the b-quark,

etc.) in searches for signs of physics beyond the standard model.

Hadronic physics involves *unstable* states and lattice techniques are being developed to study these as we have summarised. These techniques have been tested against experiment for $\rho \to \pi\pi$ and $b_1 \to \pi\omega$. In particular, we presented evidence that the spin exotic hybrid meson (made of light quarks) is at a mass around 2 GeV and has a wide width. We discussed scalar mesons, and presented evidence that the $a_0(980)$-meson is basically a $q\bar{q}$ state and that the $B_s(0^+)$-meson is also predominantly a $b\bar{q}$ state.

There is a lot to be learnt from the lattice beyond mass spectra.

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